

- 1.(15 pts)Let $A = \begin{bmatrix} -3+4i & 4+3i \\ 2-i & 1+2i \end{bmatrix}$, calculate $\|A\|_1$, $\|A\|_2$ and $\|A\|_\infty$.
- 2.(15 pts)Given $u = [1, 3]^T$. Calculate $\oint_{|\xi|=100} (1+3\xi+2\xi^2)(\xi I_2 - uu^T)^{-1} d\xi$
- 3.(15 pts)Construct a matrix $A \in \mathbb{R}^{n \times n}$ such that A^2 is symmetric, while A not.
- 4.(15 pts)Given linear transformation $T: v \rightarrow v (v \in \mathbb{R}^n)$ which satisfies $T^{n-1} \neq 0$ and $T^n = 0$, prove that $\exists x \in v$ such that $x, T(x), \dots, T^{n-1}(x)$ are linearly independent.
- 5.(15 pts) A and B are nonsingular matrices. Prove that $B^{-1} - A^{-1} = A^{-1}(A - B)A^{-1} + A^{-1}(A - B)B^{-1}(A - B)A^{-1}$
- 6.(15 pts)Given $A, B \in \mathbb{C}^{n \times n}$. A is negative finite and B is oblique Hermitite. Prove that there exists $X \in \mathbb{C}^{n \times n}$, let X^*AX and X^*BX be diagonal.
- 7.(15 pts)Prove that $\text{adj}(A)$ can be represented as the polynomial of A .
- 8.(15 pts)Known $A \in \mathbb{C}^{n \times n}$. Prove that: $\rho(A) < 1 \Leftrightarrow \exists$ Hermite Q , satisfies $Q - A^*QA \succ 0$.