# Advanced linear algebra final exam

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1.Is there exists a real matrix $A$ such that $A^4 + I = 0$ ?

2.Let $x, y \in \mathbb{C}^n$ and $y^*x \neq 0$.Proof: $\|I_n - \frac{xy^*}{y^*x}\|_2 = \frac{\|x\|_2\|y\|_2}{|y^*x|}$

3.Let $A \in \mathbb{C}^{n\times n}$ and rank$(A)$+rank$(I-A)$=$n$.Proof that $A$ is diagonalizable.

4.Let $A \in \mathbb{C}^{n\times n}$ and $v \in \mathbb{C}^n$.Proof the inequality: $\lambda_i(A) \leq \lambda_i(A + vv^*) \leq \lambda_{i+1}(A)$ $\forall i, 1 \leq i \leq n-1$.

5.Let $A$ and $E \in \mathbb{C}^{n\times n}$.Proof exp$(A+E)$-exp$(A)$=$\int_0^1 exp((1-s)A) \cdot E \cdot exp(s(A+E))\ ds$

6.Let $A, B \in \mathbb{R}^{n\times n}$ and $0 < A < B$.Proof that $\rho(A) < \rho(B)$.

7.In $\Delta ABC$,there exisit P in edge AC and Q in edge AB,which satisifies that $|BP||CQ| = |AP||AQ|$.CP and BQ converges within the triangle in $M$.Find the position of P,Q to maximize $\dfrac{S_{\Delta MBC}}{S_{\Delta ABC}}$.

8.Let $A \in \mathbb{C}^{n\times n}$.Proof that A can be represented by a finite sequence of unitary matrix.